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SUBJECT: The Shape of a Lunar Crater Ray
Case 340

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FROM: P. Gunther
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ABSTRACT

The shape of a lunar ray is influenced by conservation of momentum, action of any expanding gas cloud, and trajectory effects. An analysis is made of the trajectory of a clump of material that spreads out uniformly, and it is shown that the resultant pattern is elongated in the radial direction. For very low trajectories the pattern becomes nearly a straight line as is observed on the moon. The degree of elongation of the ray also permits one to draw inferences about the position in the crater from which the ray material originated.

MEMORANDUM FOR FILE

INTRODUCTION

During the last Science and Technology Advisory
Committee Meeting at Ames Research Center, Dr. Luis Alvarez
asked why lunar crater ray patterns are elongated in the
radial direction. Three reasons come to mind.

1. Primary ejected material will have considerable
momentum in the radial direction which tend to cause
secondary ejecta to impact in the direction of the initial
momentum and which in turn tend to form radial crater chains.

2. If the initial cratering event is accompanied by an
expanding gas cloud, dust ejected around a secondary crater
formed by a projectile will be blown away from the primary
crater leaving an elongated ray pattern in the radial direc-
tion.

3. If the initial impact ejects clumps of material (Figure 1)
which then spread out uniformly (isotropic velocity distribu-
tion of the fastest particles relative to the center of mass),
then, as shown below, the ratio of the radial axis of the
ejecta pattern to the transverse axis is $\sim 1/\sin \theta$, where θ is
the ejection angle of the center of mass. Since $1/\sin \theta > 1$,
the material will fall in a pattern which is always elongated
in the radial direction.

This memorandum examines this third case and derives
the ratio of the radial to transverse axes of the ejected
pattern.

DERIVATION

The analysis below assumes a flat moon. This would
apply to ray patterns which are within the vicinity of the
parent crater. Global patterns like those from Tycho and
Copernicus require the curved moon analysis given in the Appendix.

Let the velocity relative to the center of mass of the
fastest particles be ΔV and for those in the plane of the center
of mass trajectory, let their ejection angle relative to the
center of mass be ϕ.

It is easy to show[1] that the time of flight of the center of mass is

$$t_{cm} = \frac{2}{g} V\sin\theta$$

and the impact point of the center of mass is at a distance X where

$$X = \frac{2}{g} V\sin\theta \cdot V\cos\theta = \frac{V^2\sin 2\theta}{g}$$

In the case of a small particle with an inplane velocity ΔV, and angle ϕ relative to the center of mass, the above equations become

$$t_i = \frac{2}{g} (V\sin\theta + \Delta V\sin\phi)$$

$$X_i = \frac{2}{g} (V\sin\theta + \Delta V\sin\phi)(V\cos\theta + \Delta V\cos\phi)$$

$$= \frac{V^2\sin 2\theta}{g} + \frac{2}{g} V\Delta V\sin(\theta + \phi) + \frac{(\Delta V)^2\sin 2\phi}{g}$$

Neglecting second order terms in ΔV^2, we find that the elongation ΔX in the radial direction is

$$\Delta X = X_i - X = \frac{2V\Delta V}{g} \sin(\theta + \phi)$$

This is a maximum when $\sin(\theta + \phi) = 1$, or $\theta + \phi = \pi/2$,

and

$$\Delta X_{max} = \frac{2V\Delta V}{g}$$

Now consider the maximum width ΔY_{max} of the ejecta pattern in the transverse direction. This is given by the horizontal motion of particles perpendicular to the plane of the trajectory of the center of mass. Such a particle will impact at a distance ΔY in the transverse direction given by

$$\Delta Y_{max} = t_i \Delta V$$

Neglecting terms of order $(\Delta V)^2$, we can replace the particle impact time t_i by the time of impact of the center of mass t_{cm} so that

$$\Delta Y_{max} = \frac{2}{g} V\sin\theta \cdot \Delta V$$

Hence, the ratio of radial to transverse elongation is

$$\frac{\Delta X_{max}}{\Delta Y_{max}} = \frac{1}{\sin \theta}$$

It should be noted that $\frac{\Delta X_{max}}{\Delta Y_{max}} \to \infty$ as $\theta \to 0$, i.e., the ejecta pattern approaches a straight line as the material is ejected in a more nearly horizontal trajectory. Material ejected vertically will form a circular pattern since the motion relative to the center of mass acts equally in all directions.

In the case of a curved moon (see Appendix), for small V, $(\Delta X)_{max}$ falls off slightly as $\theta \to 0$. But as V increases to orbital velocity, $(\Delta X)_{max}$ varies as cot θ while $(\Delta Y)_{max}$ becomes constant, again resulting in an extremely elongated ray pattern.

Given an actual ray pattern in a lunar photograph, the ejection angle of the center of mass of that particular clump can be calculated from $\frac{\Delta X_{max}}{\Delta Y_{max}} = \frac{1}{\sin \theta}$. θ can then be substituted into the distance equation

$$X = \frac{V^2 \sin 2\theta}{g}$$

and the velocity of the center of mass can be found. If one can now find a way to relate the ejection angle and velocity to the source of the material, one may be able to state from what depth the material came. This is discussed below.

DISCUSSION

If one assumes a definite relationship between the ejection velocity and the distance that a particular chunk was displaced from the center of the cratering event, or even only that the velocity monotonically decreases with distance from the center of pressure, then one can draw certain conclusions about the origin of the material in the ray pattern.* (See Figure 1.)

Case 1. $\frac{\Delta X}{\Delta Y} = \sqrt{2}$

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*The trajectory considerations underlying the conclusions are discussed in Reference 1.

Here θ = 45° which corresponds to a minimum energy trajectory for a given radial distance X. The material has the lowest velocity and probably came from the surface of the originally impacted region.

Case 2. $\dfrac{\Delta X}{\Delta Y} \gg 1$

Here $\theta \rightarrow 0$ and the velocity reaches its maximum value for low angle trajectories intersecting the moon. The material probably came from close to the center of pressure.

Case 3. $\dfrac{\Delta X}{\Delta Y} = 1$

Here $\theta \rightarrow 90°$ and the velocity again reaches high values (close to but less than escape velocity) with the material coming from close to the center of pressure--probably deeper than Case 2 since the crater walls would not interfere with ballistic trajectories.

Any analysis of this type should be wary of the other effects mentioned in the introduction. If the ejecta pattern is caused by low angle impacts with tertiary ejecta being ejected forward to form a chain of radial craters, one could come up with a pseudo $\dfrac{\Delta X}{\Delta Y} \gg 1$. But in this case $\theta \rightarrow 0$ and V is also high so that the confusion of causes does not invalidate the hypotheses of the depth of origin of the material.

On the other hand, if the ejecta pattern is caused by a steep and hence high velocity impact at distance X with the subsequent blowing of any secondary debris out in a radial direction by any gas expanding from the primary impact, then a pattern approaching $\dfrac{\Delta X}{\Delta y} = \sqrt{2}$ could be formed. This would lead to confusion between high velocity and low velocity ejecta. To avoid this ambiguity one should carefully look for a head crater at the primary crater end of the ejecta pattern. These typically have V-shaped dune patterns and can probably be eliminated.*

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Attachments
 Reference
 Figure 1
 Appendix

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*F. El-Baz, private communication.

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REFERENCE

1. G. K. Chang, P. Gunther, and D. B. James, "A Secondary Ejecta
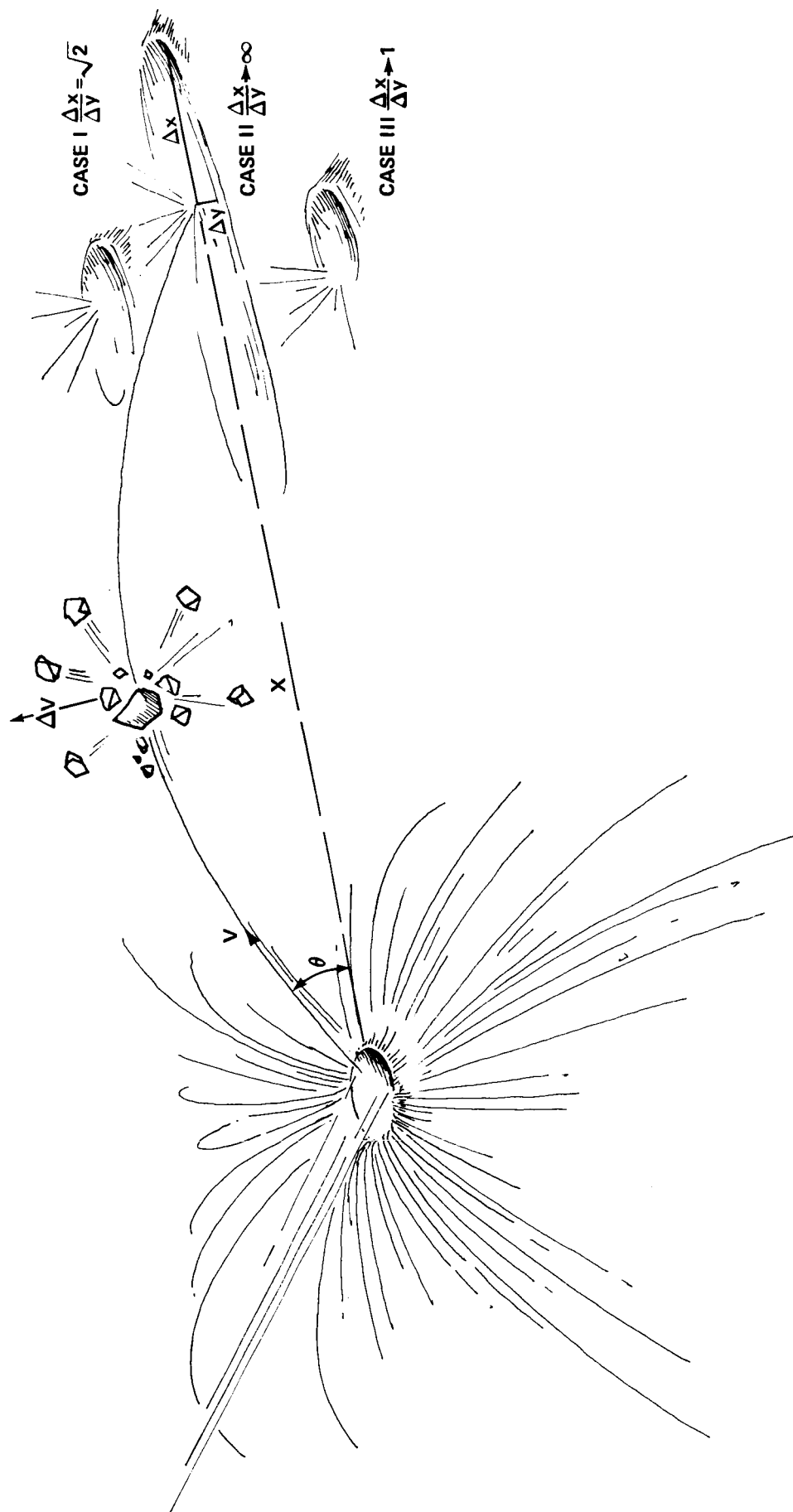   Explanation of a Lunar Seismogram," Bellcomm TM70-2015-2,
    March 17, 1970.

CASE I $\frac{\Delta x}{\Delta y} = \sqrt{2}$

CASE II $\frac{\Delta x}{\Delta y} \to \infty$

CASE III $\frac{\Delta x}{\Delta y} \to 1$

FIGURE 1 - FORMATION OF RAY PATTERN FROM EJECTED CLUMP

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APPENDIX

CURVED MOON ANALYSIS

Consider an individual particle whose velocity vector $\overrightarrow{\Delta V}$, relative to the center of mass, has direction ψ in azimuth and ϕ in elevation. We wish to determine the perturbation in the impact point of the particle from the center of mass impact.

Let δV and $\delta\theta$ represent the perturbations in V and θ due to $\overrightarrow{\Delta V}$. To order $(\Delta V)^2$, δV is given by the projection of $\overrightarrow{\Delta V}$ onto $\vec{V}$, namely

$$\delta V = \Delta V \ (\cos \theta \ \cos \phi \ \cos \psi + \sin \theta \ \sin \phi) \qquad (1)$$

while $\delta\theta$ is derived from the projection of $\overrightarrow{\Delta V}$ onto the (orbital) plane containing $\vec{V}$ which leads to

$$\delta\theta = \frac{\Delta V}{V} \ (\cos \theta \ \sin \phi - \sin \theta \ \cos \phi \ \cos \psi) \qquad (2)$$

Now the central angle F traversed by the center of mass is given by[1]

$$F = 2 \ \tan^{-1} \left(\frac{\tan \theta}{1-V^2/V_o^2} \right) - 2\theta \qquad (3)$$

where V_o is the circular orbit velocity at the surface of the moon. The differential of (3) gives the down-range perturbation

$$\Delta F = \frac{2}{\left[(1-V^2/V_o^2)^2 + \tan^2\theta\right]} \left[\frac{2V \ \tan \theta}{V_o^2} \ \delta V + \left(1-\frac{V^2}{V_o^2}\right) \sec^2\theta \cdot \delta\theta \right] - 2 \ \delta\theta \qquad (4)$$

Substituting (1) and (2) into (4) yields

$$\Delta F = \frac{2V \cdot \Delta V}{V_o^2 \left\{(1-V^2/V_o^2)^2 + \tan^2 \theta\right\}} \Big\{ 2 \tan \theta \ (\cos \theta \ \cos \phi \ \cos \psi + \sin \theta \ \sin \phi) + \left(1-\frac{V^2}{V_o^2} - \tan^2\theta\right) (\cos \theta \ \sin \phi - \sin \theta \ \cos \phi \ \cos \psi) \Big\} \qquad (5)$$

Consider now the maximum of (5) with respect to ϕ and ψ. For fixed ϕ, (5) is a maximum when $\psi = 0$, since the coefficient of $\cos \psi$ is proportional to $2 - \left(1 - \tan^2\theta - v^2/v_o^2\right) = \sec^2\theta + v^2/v_o^2 > 0$. If we define the angle α by

$$\alpha = \tan^{-1} \left(\frac{1 - \tan^2\theta - v^2/v_o^2}{2 \tan \theta} \right) \tag{6}$$

then for $\psi = 0$ the expression in braces in (5) is proportional to $\cos(\phi - \theta - \alpha)$. Hence when $\phi = \theta + \alpha$ one gets the maximum value

$$(\Delta F)_{max} = \frac{2V \cdot \Delta V}{v_o^2} \cdot \frac{\left[4 \tan^2\theta + \left(1 - \tan^2\theta - v^2/v_o^2\right)^2\right]^{1/2}}{1 - v^2/v_o^2 + \tan^2\theta}$$

$$= \frac{2V \cdot \Delta V}{v_o^2} \cdot \frac{\left[1 - 2\left(v^2/v_o^2\right) \cos^2\theta \cos 2\theta + v^4/v_o^4 \cos^4\theta\right]^{1/2}}{1 - 2\left(v^2/v_o^2\right) \cos^2\theta + \left(v^4/v_o^4\right) \cos^2\theta} \tag{8}$$

When $V \ll V_o$, the right-hand factor is approximately unity. Since $(\Delta X)_{max} \underset{\sim}{=} (\Delta F)_{max} \cdot R_o$, where R_o is the radius of the moon, and $g = v_o^2/R_o$, one gets the flat moon solution. More precisely, for V/V_o small, a Maclaurin expansion leads to

$$(\Delta X)_{max} \underset{\sim}{=} \frac{2V \cdot \Delta V}{g} \left[1 + \frac{v^2}{v_o^2} (1 + \sin^2\theta \cos 2\theta)\right] \tag{9}$$

When $V = V_o$, (7) becomes

$$(\Delta F)_{max} = \frac{2\Delta V}{V_o} \sqrt{1 + 4 \cot^2\delta} \tag{10}$$

For $\theta = 0$, (10) becomes infinite--actually, V_o incremented by ΔV leads to a non-intersecting elliptic orbit which periodically returns to the original point of ejection. For small θ, $(\Delta F)_{max}$ is proportional to $\cot \theta$.

More generally, for given V the ejection angle, say $\hat{\theta}$, for which (7) is a maximum is

$$\tan \hat{\theta} = \frac{1-v^2/v_o^2}{\sqrt{3-v^2/v_o^2}} \tag{11}$$

and the corresponding $(\Delta F)_{max}$ is

$$(\Delta \hat{F})_{max} = \frac{4V \cdot \Delta V}{v_o^2} \cdot \frac{1}{\left(1-v^2/v_o^2\right)\sqrt{4-v^2/v_o^2}} \tag{12}$$

Note that $\hat{\theta}$ decreases from 30° when $V = 0$ (compare also (9)) to 0° when $V = V_o$.

The transverse perturbation ΔY of the particle is, to order $(\Delta V)^2$, simply the transverse component of $\overrightarrow{\Delta V}$ multiplied by time of flight, i.e.,

$$\Delta Y = \Delta V \cos \phi \sin \psi \cdot t_{cm} \tag{13}$$

This is a maximum when $\psi = 90°$ and $\phi = 0$, so that

$$(\Delta Y)_{max} = \Delta V \cdot t_{cm} \tag{14}$$

t_{cm}, obtained from Kepler's equation, can be written[1]

$$t_{cm} = \frac{2V_o}{g} \left(2-\frac{v^2}{v_o^2}\right)^{-3/2} \left[\tan^{-1}\left\{\frac{V \sin \theta}{V_o}\sqrt{\frac{2-v^2/v_o^2}{1-v^2/v_o^2}}\right\} + \frac{V \sin \theta}{V_o}\sqrt{2-v^2/v_o^2}\right] \tag{15}$$

When $V = V_o$, (15) reduces to

$$t_{cm} = \frac{2V_o}{g} \left(\frac{\pi}{2} + \sin \theta\right) \tag{16}$$

except possibly when $\theta = 0$. When $V << V_o$, (15) approaches the flat moon solution. More precisely, for V/V_o small a Maclaurin expansion leads to the approximation

$$t_{cm} \cong \frac{2V \sin \theta}{g} \left(1 + \frac{3v^2}{4v_o^2}\right) \tag{17}$$

Combining (9), (14) and (17) gives the following approximation when $V << V_o$

$$\frac{(\Delta X)_{max}}{(\Delta Y)_{max}} \approx \frac{1}{\sin \theta} \cdot \frac{1 + \frac{V^2}{V_o^2}\left(1 + \sin^2\theta \, \cos \, 2\theta\right)}{1 + \frac{3}{4} \cdot \frac{V^2}{V_o^2}} \qquad (18)$$

$$\approx \frac{1}{\sin \theta} \left[1 + \frac{V^2}{V_o^2}\left(\frac{1}{4} + \sin^2\theta \, \cos \, 2\theta\right)\right] \qquad (19)$$

Similarly, when $V = V_o$, equations (10), (16) and (14) yield

$$\frac{R_o (\Delta F)_{max}}{(\Delta Y)_{max}} = \frac{\sqrt{1 + 4 \, \cot^2\theta}}{\frac{\pi}{2} + \sin \, \theta} \qquad (20)$$

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